Name: Solutions Section: ________

1. (a) Write the following augmented matrix as a vector equation, a matrix equation, and a system of linear equations.

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -3 \\ 2 & 3 & 1 & 1 \end{array}\right]$$

$$x_1 \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_2 \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} + x_3 \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$\begin{cases} x_1 + 2x_2 - x_3 = -3 \\ 2x_1 + 3x_2 + x_3 = 1 \end{cases}$$

3 cases: 2pt for all 3; 1pt for 2; 0.5 pt for 1.

(b) Is the system of equations consistent?

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & -3 \\ 0 & -1 & 3 & 7 \end{array}\right] r_2 - 2r_1$$

(1pt)

← echelon form doesn't contain $[0 \dots 0 \mid \blacksquare]$ $\Rightarrow$ consistent (1pt)

(c) Solve the above system. (Find the solution set).

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 5 & 11 \\ 0 & -1 & 3 & 7 \end{array}\right]^{r_1 + 2r_2} \sim \left[\begin{array}{ccc|c} 1 & 0 & 5 & 11 \\ 0 & 1 & -3 & -7 \end{array}\right]$$

(2pt) for final matrix

$$\Longleftrightarrow \begin{cases} x_1 + 5x_3 = 11 \\ x_2 - 3x_3 = -7 \\ x_3 \text{ free} \end{cases} \Longleftrightarrow \begin{cases} x_1 = 11 - 5x_3 \\ x_2 = -7 + 3x_3 \\ x_3 \text{ free} \end{cases}$$

2pt for parametric form

(d) If the system is consistent, write down a particular solution.

Verify that this is a solution *two different ways* by plugging it into both *i.* the vector equation, and *ii.* the system of equation.

Pick $x_3 = 0$

$$\Rightarrow \vec{x} = \begin{bmatrix} 11 \\ -7 \\ 0 \end{bmatrix}$$

$$11 \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} + (-7)\begin{bmatrix} 2 \\ 3 \end{bmatrix} + 0 \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 11 \\ 22 \end{bmatrix} + \begin{bmatrix} -14 \\ -21 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix} \checkmark$$

$$\begin{cases} 11 + 2 \cdot (-7) - 0 = 11 - 14 = -3 \checkmark \\ 2 \cdot 11 + 3 \cdot (-7) + 0 = 22 - 21 = 1 \checkmark \end{cases}$$

1pt each

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2. (a) Write the following augmented matrix as a vector equation, a matrix equation, and a system of linear equations.

$$\left[\begin{array}{cc|c} 0 & 4 & 6 \\ 5 & 2 & 10 \\ 3 & 3 & 6 \end{array}\right]$$

$$x_1 \cdot \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix} + x_2 \cdot \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 4 \\ 5 & 2 \\ 3 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 6 \end{bmatrix}$$

$$\begin{cases} 4x_2 = 6 \\ 5x_1 + 2x_2 = 10 \\ 3x_1 + 3x_2 = 6 \end{cases}$$

(b) Is the system of equations consistent?

$$\sim \left[\begin{array}{cc|c} 3 & 3 & 6 \\ 5 & 2 & 10 \\ 0 & 4 & 6 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 5 & 2 & 10 \\ 0 & 2 & 3 \end{array}\right] \begin{matrix} \tfrac{1}{3}r_1 \\ \\ \tfrac{1}{2}r_3 \end{matrix} \sim \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -3 & 0 \\ 0 & 2 & 3 \end{array}\right] r_2 - 5r_1$$

$$\sim \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 2 & 3 \end{array}\right] -\tfrac{1}{3}r_2 \sim \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{array}\right] r_3 - 2r_1$$

requires $0 = 3$

INCONSISTENT!

(c) Solve the above system. (Find the solution set).

No solutions

(empty set)

(d) If the system is consistent, write down a particular solution.

Verify that this is a solution *three different ways* by plugging it into *i.* the vector equation, *ii.* matrix equation, and *iii.* system of equation.

N/A

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3. (a) Write the following augmented matrix as a vector equation, a matrix equation, and a system of linear equations.

$$\left[\begin{array}{ccc|c} 3 & -3 & 3 & 3 \\ -1 & 2 & 1 & 3 \\ 2 & 8 & 2 & 2 \end{array}\right]$$

$$x_1 \cdot \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} + x_2 \cdot \begin{bmatrix} -3 \\ 2 \\ 8 \end{bmatrix} + x_3 \cdot \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$$

$$\begin{cases} 3x_1 - 3x_2 + 3x_3 = 3 \\ -x_1 + 2x_2 + x_3 = 3 \\ 2x_1 + 8x_2 + 2x_3 = 2 \end{cases}$$

$$\begin{bmatrix} 3 & -3 & 3 \\ -1 & 2 & 1 \\ 2 & 8 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$$

(b) Is the system of equations consistent?

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ -1 & 2 & 1 & 3 \\ 1 & 4 & 1 & 1 \end{array}\right] \begin{array}{l} \tfrac{1}{3}r_1 \\ \\ \tfrac{1}{2}r_2 \end{array} \sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & 2 & 4 \\ 0 & 5 & 0 & 0 \end{array}\right] \begin{array}{l} \\ r_2 + r_1 \\ r_3 - r_1 \end{array}$$

Step form

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 1 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 4 \end{array}\right]$$

consistent because omits $[0\ 0\ 0 \mid \blacksquare$

(c) Solve the above system. (Find the solution set).

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} r_1 + r_2 \\ \\ \tfrac{1}{2}r_3 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} r_1 - r_3 \\ \\ \\ \end{array}$$

$$\begin{cases} x_1 = -1 \\ x_2 = 0 \\ x_3 = 2 \end{cases}$$

(d) If the system is consistent, write down a particular solution.

Verify that this is a solution *three different ways* by plugging it into *i.* the vector equation, *ii.* matrix equation, and *iii.* system of equation.

~~vector~~ equation
Vector

$$-1 \cdot \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} + 0 \cdot \begin{bmatrix} -3 \\ 2 \\ 8 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} \checkmark$$

system of eqns

$$\begin{cases} 3 \cdot (-1) + -3 \cdot 0 + 3 \cdot 2 = -3 + 6 = 3 \checkmark \\ -1(-1) + 2 \cdot 0 + 1 \cdot 2 = 1 + 2 = 3 \checkmark \\ 2(-1) + 8 \cdot 0 + 2 \cdot 2 = -2 + 4 = 2 \checkmark \end{cases}$$